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A Conjecture for possible theory for the description of high temperature superconductivity and antiferromagnetism

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Abstract

Keeping in mind some salient features of quantum symmetry groups and in lieu of the interesting critique by Baskaran and Anderson of the hypothesis of the $SO(5)$ model of Zhang and the related works of others such as Sachdev to describe superconductivity and antiferromagnetism we propose a conjecture to model these complex phenomena by quantum groups, at least as a starting point.

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The Hubbard Hamiltonian [HH] and its extensions dominate the study of strongly correlated electrons systems and the insulator metal transition [1]. One of the attractive feature of the Hubbard Model is its simplicity. It is well known that in the HH the band electrons interact via a two-body repulsive Coulomb interaction; there are no phonons in this model and neither in general are attractive interactions incorporated. With these points in mind it is not surprising that the HH was mainly used to study magnetism. In contrast superconductivity was understood mainly in light of the BCS theory, namely as an instability of the vacuum [ground-state] arising from effectively attractive interactions between electron and phonons. However Anderson [2] suggested that the superconductivity in high T_c material could arise from purely repulsive interaction. The rationale of this suggestion is grounded in the observation that superconductivity in such materials arises from the doping of an otherwise insulating state. Thus following this suggestion the electronic properties in such a high T_c superconductor material close to a insulator-metal transition must be considered. In particular the one-dimensional HH is considered to be the most simple model which can account for the main properties of strongly correlated electron systems including the metal-insulator transition. Long range antiferromagnetic order at half-filling has been reported in the numerical studies of this model [3,4]. Away from half-filling this model has been studied in [5].

Zhang [6] proposed a unified theory of superconductivity and antiferromagnetism [†]based on $SO(5)$ symmetry and suggested that there exists an approximate global $SO(5)$ symmetry in the low temperature phase of the high T_c cuprates. In this model one has a five component order parameter. Three components correspond to a spin one, charge zero

[†]We note that theories of cuprates based upon a quantum critical point have also been suggested by others, see for e.g. Sachdev et al. [8].

particle-hole pair condensed at the center of mass momentum (π, π) , these components represent antiferromagnetic order in the middle of Mott insulating state. The remaining two components correspond to a spin-singlet charge $\pm 2e$ Cooper pair of orbital symmetry $d_{x^2-y^2}$ condensed in zero momentum state, these last two components are supposed to correspond to superconductivity in the doped Mott insulator.

Baskaran and Anderson [7] have presented an elegant series of criticisms of the work in ref. [6]. In our opinion these criticisms have been stated so well that it is difficult to improve on these, so whenever we refer to these we stay with the original wording, almost as is presented in [7]. We first state the observation in [7] that is of immediate interest to us:

- The antiferromagnetic and superconducting phases derive from more fundamental phases, namely from the Mott insulator and metal respectively. Now comes the key observation, namely: “The Mott insulator and the metal cannot be related to a **quantum critical point**, since they differ by a local gauge symmetry. These phases hence have no locally stable, homogeneous intermediate phases, and cannot be deformed continuously into each other, certainly not by an operator as simple as an $SO(5)$ rotation.”

In the elaboration of this point, Baskaran and Anderson [7] forcefully make the point that theories of cuprates which are based on the existence of *quantum critical point* cannot realistically describe these systems since there is no underlying quantum critical point expressing an essential continuity between the two phases. For further details of this point and others raised by Baskaran and Anderson [7], we refer the reader to their paper.

We now recall some useful details about quantum groups. A quantum group can be considered as a deformation of the classical Lie group. If we introduce a parameter q , proportional \hbar we may express this the idea of this deformation as the statement

$$\lim_{q \rightarrow 1} \text{Quantum Group} = \text{Classical Lie Group.} \quad (1)$$

It is interesting to note some important differences between a quantum group and an ordinary classical group [9]

- The braiding operations differ between the two types of groups. In the case of a classical group the braiding operations simply interchanges the representations forming the tensor product of two representations leading to +1 for a symmetric interchange and -1 for antisymmetric interchange of the representation. However for a quantum group the braiding operation picks up crucial *phase* factors. Indeed the presence of these nontrivial phases encourages us to consider our proposal seriously.
- One finds the relationship

$$q \leftrightarrow e^{2\pi i/(k+2)}$$

between the q found in quantum groups and k appearing in Kac-Moody algebra, while calculating the “Clebsch-Gordon” coefficients generated by these algebras [9]. This relationship arises due to an important reason: unlike ordinary Lie algebras, Kac-Moody algebra contain central charges c , thus even when one reduces Kac-Moody algebra, the reduction process leads not to ordinary Lie algebras but to something more general, i.e. quantum groups.

In view of the critique presented in ref. [7], in particular the observation of Baskaran and Anderson presented above and the features of quantum groups we are led to our conjecture: At the simplest level and as a preliminary step, it is tempting to base a model for superconductivity and antiferromagnetism on a quantum group symmetry rather than the usual classical Lie group. We consider this as a preliminary step, since it is quite likely that a realistic model which unifies a complex system containing antiferromagnetic and superconducting phases, may require the mathematical machinery currently being used for string

theory, or something even beyond it. Another important motivation for our conjecture is to model Stripes. Stripes can be aptly described as being found in the unstable two-phase region between the antiferromagnetic and metallic states [10]. One of the real challenges is to formulate a theory which gives rise to the equivalent of “Fermi surface”[‡] whose excitation surface derives from *fluctuations that are not uniform in space*. Intuitively one may imagine these non-uniform fluctuations as arising out of a nonlinear sigma-like model. We further conjecture that superconductivity arises when two immiscible phases, namely a 2-D antiferromagnetic state and a 3-D metallic state, are “forced” to meet at $T_c \rightarrow \infty$. As is well-known, ordinary low temperature superconductors arise entirely out of a “metallic” like state. In contrast High T_c superconductors have relatively large T_c since we *cannot* smoothly map two immiscible states [metallic and antiferromagnetic] together. It is the lack of this smooth mapping that is precisely responsible for the High T_c . The lack of smooth mappings may be modelled by the nontrivial phases which arise out of the braiding operations of quantum groups.

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[‡]We mean, as is usual [10], that with a metallic state we can always associate a “Fermi surface” which describes the low-energy excitations. This is a finite volume surface in momentum space which arises due to all the one-particle amplitudes [10].

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